

EE 508
Homework 2
Fall 2022
Due Friday Sept 16

Problem 1 Obtain all approximating functions with real coefficients that agree in the magnitude squared sense with one with poles at $-1+j1$, $-1-j1$, $-4+j8$, $-4-j8$ and with zeros at 2 , $2+j6$ and $2-j6$

Problem 2 Which of those approximating functions in Problem 1 are minimum phase?

Problem 3 Some magnitude-squared functions are given below. If a function $T(s)$ exists that agrees with these functions in the magnitude-squared sense, determine it.

a)
$$H(\omega^2) = \frac{1 + 2\omega^2 + 6\omega^4}{\omega^8 + 2\omega^6 + 3\omega^4 + 2\omega^2 + 16}$$

b)
$$H(\omega^2) = \frac{1 + 4\omega^4}{\omega^6 + 2\omega^4 + 2\omega^2 + 1}$$

c)
$$H(\omega^2) = \frac{1 + \omega + 2\omega^2}{\omega^4 + 2\omega^2 + 1}$$

d) $H(\omega^2)$ has poles at $-1+j1$, $-1-j1$, $1+j1$, $1-j1$, and a double pole at $2+j0$ and zeros at $-2+j6$, $2-j6$, $2+j6$, $-2-j6$ and a double zero at $-4+j0$

e) $H(\omega^2)$ has poles at $-1+j1$, $-1-j1$, $1+j1$, $1-j1$, $-2+j0$ and zeros at $-2+j6$, $2-j6$, $2+j6$, $-2-j6$ and a double zero at $-4+j0$

Problem 4 Prove or disprove the following theorem:

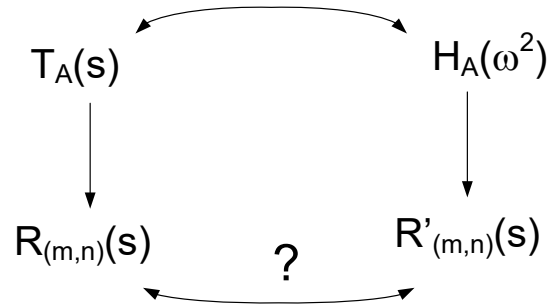
Theorem: If $H_A(\omega^2)$ is the magnitude squared approximation of the all-pole transfer function $T(s)$, then the coefficients of $H_A(\omega^2)$ are all of the same sign..

Problem 5 Obtain the 2,3 order Pade approximation $T_{P23}(s)$ of the rational fraction

$$T(s) = \frac{1}{s^5 + 3.2s^4 + 5.3s^3 + 5.3s^2 + 3.2s + 1}$$

and comment on how good the Pade approximation is by comparing the magnitude response of $T(s)$ and $T_{P32}(s)$.

Problem 6 If $R_{(m,n)}(s)$ is the Pade approximation of $T_A(s)$ and $R'_{(m,n)}(s)$ is the Pade approximation of $H_A(\omega^2)$ where $H_A(\omega^2)$ is the magnitude squared function corresponding to transfer function $T_A(s)$, are $R_{(m,n)}(s)$ and $R'_{(m,n)}(s)$ equal? Either prove this relationship or show why it is not true.



Problem 7 $T(s) = \frac{Ks \frac{\omega_0}{Q}}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ is the basic second-order bandpass

transfer function. Determine the two 3-dB band edges and prove that

- The magnitude of the gain is a maximum at $\omega = \omega_0$
- The maximum gain magnitude is K
- The 3dB bandwidth is ω_0/Q
- The geometric mean of the two 3-dB band edges is ω_0

Problem 8

- What is the minimum value of Q of a pole pair that will result in complex conjugate pole pairs?
- Derive an expression for the angle of a complex pole relative to the imaginary axis in the s -plane in terms of the parameters ω_0 and Q

Problem 9

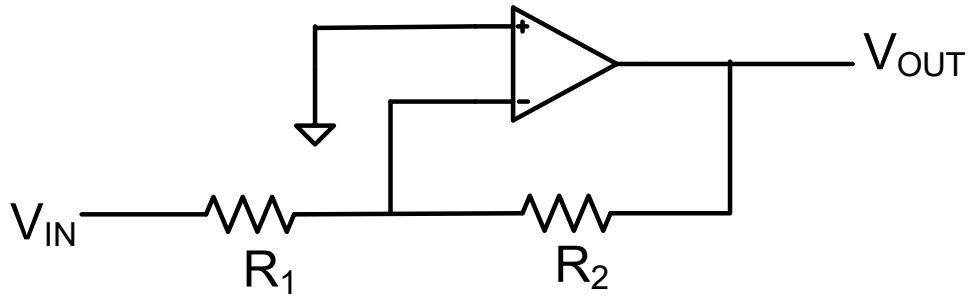
a) Using collocation, determine the 6-th order magnitude-squared approximating function with a fourth-order numerator polynomial that has collocation points $(0,0.00625), (1,0.00717), (2.5,0.748), (3.5,2.838), (4.5,0.860), (8,0.169)$

If it exists, determine the corresponding transfer function $T_A(s)$.

b) Repeat part a) but assume the third point is changed to $(2.5,1.0)$ Comment on the sensitivity to the individual component values

Problem 10

Consider the basic inverting amplifier where the op amp is modeled with a gain function $A(s) = \frac{GB}{s}$. Define $K_0 = 1 + \frac{R_2}{R_1}$.



- Derive an expression for the frequency dependent gain in terms of the model parameters GB and K_0 .
- Derive an expression for the closed-loop gain bandwidth product in terms of GB and K_0 .

Problem 11. Consider the two designs of a finite gain amplifier shown below.

- Determine resistor values so that the gain of the two designs is 36
- If the op amps are modeled with a gain $A(s) = \frac{GB}{s}$ and $GB=1\text{MHz}$, derive an expression for the 3dB bandwidth of the two designs and comment on the relative performance.

